

Examen de Matemáticas 1º de Bachillerato

Problema 1 Dados los números complejos $z_1 = 5 - 2i$ y $z_2 = -1 + 3i$. Se pide calcular:

a) $z_1 + z_2$ y $z_1 - z_2$

b) $z_1 \cdot z_2$

c) $\frac{z_1}{z_2}$

Solución:

a) $z_1 + z_2 = 4 + i$ y $z_1 - z_2 = 6 - 5i$

b) $z_1 \cdot z_2 = 1 + 17i$

c) $\frac{z_1}{z_2} = -\frac{11}{10} - \frac{17}{10}i$

Problema 2 Resolver la siguiente ecuación de segundo grado:

$$z^2 + 3z + 5 = 0$$

Solución:

$$z^2 + 3z + 5 = 0 \implies z = \begin{cases} -\frac{3}{2} + \frac{\sqrt{11}}{2}i \\ -\frac{3}{2} - \frac{\sqrt{11}}{2}i \end{cases}$$

Problema 3 Si $z = 3 - 7i$ calcular z^{10} .

Solución:

$$z = 3 - 7i = \sqrt{58}_{293^\circ 11' 55''} = \sqrt{58}(\cos 293^\circ 11' 55'' + i \sin 293^\circ 11' 55'')$$

$$z^{10} = (3 - 7i)^{10} = 58^5_{10 \cdot 293^\circ 11' 55''} = 58^5_{2931^\circ 59' 10''} = 58^5_{51^\circ 59' 10''} = 58^5(\cos 51^\circ 59' 10'' + i \sin 51^\circ 59' 10'')$$

Problema 4 Resolver la ecuación $z^3 - i = 7$.

Solución:

$$z^3 = 7 + i \implies z = \sqrt[3]{7 + i}$$

$$7 + i = 5\sqrt{2}_{8^\circ 7' 48''} = 5\sqrt{2}(\cos 8^\circ 7' 48'' + i \sin 8^\circ 7' 48'')$$

$$z = \sqrt[3]{7 + i} = \begin{cases} \sqrt[6]{50}_{2^\circ 42' 36''} = \sqrt[6]{50}(\cos 2^\circ 42' 36'' + i \sin 2^\circ 42' 36'') \\ \sqrt[6]{50}_{122^\circ 42' 36''} = \sqrt[6]{50}(\cos 122^\circ 42' 36'' + i \sin 122^\circ 42' 36'') \\ \sqrt[6]{50}_{242^\circ 42' 36''} = \sqrt[6]{50}(\cos 242^\circ 42' 36'' + i \sin 242^\circ 42' 36'') \end{cases}$$

Problema 5 Calcular las raíces de $\sqrt[3]{-2+3i}$

Solución:

$$z = -2 + 3i = \sqrt{13}_{123^{\circ}41'24''} = \sqrt{13}(\cos 123^{\circ}41'24'' + i \sin 123^{\circ}41'24'')$$

$$\sqrt[3]{z} = \begin{cases} \sqrt[6]{13}_{41^{\circ}13'48''} = \sqrt[6]{13}(\cos 41^{\circ}13'48'' + i \sin 41^{\circ}13'48'') \\ \sqrt[6]{13}_{161^{\circ}13'48''} = \sqrt[6]{13}(\cos 161^{\circ}13'48'' + i \sin 161^{\circ}13'48'') \\ \sqrt[6]{13}_{281^{\circ}13'48''} = \sqrt[6]{13}(\cos 281^{\circ}13'48'' + i \sin 281^{\circ}13'48'') \end{cases}$$